THE JOST-SCHROER THEOREM AND WILSON's FIXED POINT CONDITION FOR THE RENORMALIZED COUPLING CONSTANT☆

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On the basis of the Jost-Schroer-Pohlmeyer-Strocchi theorems it is proved that Wilson's renormalization group approach leads to the fixed point condition $\alpha_\Lambda^{\mathrm{fix}} = 0$, unless the skeleton theory is a nonlocal theory with a non-analytic dependence on mass and coupling parameter. This result resolves a paradoxon in connection with the Adler-Schwinger-Bell-Jackiw anomaly.

The main assumption in Wilson's [1] renormalization group approach is that one can subdivide an enormous cutoff Λ of the order of 10^{20} GeV in such a way that finally one ends up with a smooth theory where Λ can be put to zero and the coupling constant α_Λ is independent of Λ. *The assumption is therefore that the theory is analytic simultaneously in* Λ *and the coupling constant* α_Λ. Accordingly, the hope is also expressed that the change in Λ and α_Λ does not destroy microcausality and furthermore it is assumed that physics is not changed by making the subdivision: $\Lambda \to \Lambda/2, \Lambda/4 \ldots \Lambda/2^n \ldots$. Now if this is the case one can use the Jost-Schroer [e.g. 2]-Pohlmeyer [3]-Stocchi [4] theorems, intuitively:

"If the two point function is equal to that of a free zero-mass theory, the field is necessarily a free field".

The theorem is based on the following assumptions, which are essential to Wilson's approach, i) locality, ii) Lorentz invariance, iii) positivity †1, iiii) unique vacuum, iiiii) finite component fields.

This theorem is directly applicable to Wilson's approach because the analyticity and uniformity assumptions follow essentially from the locality postulate and the existence of a unique vacuum. Therefore if the theory is really analytic and smooth against a variation of Λ to the value zero, the variation of α_Λ – which certainly depends on Λ, as stressed also by Wilson [1] – must be also smooth. But if α_Λ can be set to zero in a smooth way, then it always has been there in the first place. Therefore we conclude

$$\alpha_\Lambda^{\mathrm{fix}} = 0. \qquad (1)$$

Before sketching our arguments we indicate the following implications of this result

1) One has to give up the locality postulate for the skeleton theory, which has been already suggested by Strocchi [4].

2) Coupling and mass are intimately related [5]. This gives then rise to a *coupling-mass relation* [6] ("charge-mass" relation), which is nothing else than a coupling as well as mass quantization condition:

3) Eq. (1) resolves a paradoxon [7] connected with the Adler-Schwinger-Bell-Jackiw (ASBJ) anomaly [8] in the socalled finite theory approach [9]: namely it was remarked that the assumption $\alpha_\Lambda^{\mathrm{fix}} \neq 0$, the non-renormalization of the ASBJ anomaly and the existence of the mass zero limit are not compatible with each other. This can be also understood in more physical terms: *a zero mass particle* (like the neutrino, or the photon) *cannot have a charge*. This follows also from the "charge-mass" relation referred to above.

4) In view of the current interest in parton models [10] or the light-cone algebra approach [11] as frameworks for studying the consequences of dilatation invariance at small distances the question arises in what sense assumptions 1) .... iiiii) are broken in these models. These models *formally* respect the locality

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†1 Strocchi ref. [4] has extended the theorem also to the case of an indefinite metric theory.

postulate, however, the assumption of the existence of a *unique* vacuum is violated †2. This follows from the Castell [15] theorem, namely, it has been shown that a spontaneously broken internal symmetry (i.e. γ_5-symmetry) implies also a spontaneously broken dilatation symmetry. Furthermore, it was pointed out that the restriction of the relevant representation to the Poincaré group leads to a *continous mass spectrum*. In this context it should be remarked also that the conformal bootstrap approaches [16] do not respect assumption iiii), as long as the wave-function renormalization "constants" vanish identically †3.

After these general remarks we now prove eq. (1). We consider the fermion propagator †4 in a Lorentz-invariant, local theory. It has the following short distance behaviour [17].

$$S(p) \approx \frac{c}{\gamma\cdot p}\left(\frac{p^2}{\Lambda^2}\right)^{(d(e_\Lambda)-3/2)} \qquad (2)$$

Now, true dilatation invariance [e.g. 18] together with locality, Lorentz-invariance and finite component fields imply the fixed point condition

$$2d(e_\Lambda) - 3 = 0, \qquad e_\Lambda = e_\Lambda^{\text{fix}} \qquad (3)$$

because a dependence on Λ^2 would mean a continuous mass spectrum and therefore an infinite †5 component field theory. However, the existence of a continuous mass representation destroys the uniqueness of the vacuum, as noted before.

Concerning eq. (3) we would like to remark first of all that this fixed point condition prohibits the smooth variation of the coupling constant away from the eigenvalue of the above equation. (In the spirit of the Wilson approach this equation cannot be *identically* [16] satisfied.)

We now even proceed to show that this fixed point condition can only be eq. (1), viz. $\alpha_\Lambda^{\text{fix}} = 0$. To obtain this we go over from the propagator to the Wightman functions, in order to use the above mentioned theorems. The ambiguity in the definition of the T-products [19] gives us the following form

$$\langle 0|\psi(x)\,\bar{\psi}(0)|0\rangle = S^+(x,0) + \mathrm{D}\,\delta(x) \qquad (4)$$

where D denotes a finite differential operator. We exclude entire analytic functions in p^2 of the lowest growth order (Jaffe type [20]) because of Wilson's smoothness assumption. Looking at the Källen-Lehmann representation we see that this term could only come from subtractions which would imply spectral measures $\rho_i(m^2)$ with unbounded support- at least. This contradicts the "free" form of the propagator. We are now ready to apply Pohlmeyer-Strocchi. Their proof depends only on support properties of the Wightman functions in p-space and is not changed by an additional factor $\gamma\cdot p$. We conclude with a positive remark concerning the Wilson approach: The idea of working with an underlying dilatation invariant skeleton theory is very interesting indeed, because it opens the possibility to interpret this infinite component nonlocal field †6 as a continuous spin [21] neutrino field.

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†2 It has been clearly pointed out by Domokos [12] that a super strong coupling theory can give rise to a "free field" behaviour of the theory, if the vacuum is completely degenerate. However, it is well known [13, 14] that an underlying degenerate vacuum implies a *non-analytic* dependence on the symmetry breaking parameters.

†3 This implies again a continuous mass spectrum.

†4 The conclusions are unchanged if we would consider the boson propagator.

†5 Note that assumption iiii) is actually not needed for our argument.

†6 The existence of such a field is already implied by the fact that in the conformal bootstrap [16] approach the constant c in eq. (2) becomes singular for an identically vanishing $2d(e_\Lambda) - 3$.

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