LETTERE AL NUOVO CIMENTO VOL. 4, N. 10 8 Luglio 1972

Does the Simple Quark Light-Cone Current Algebra of Fritzsch and Gell-Mann Imply that Quarks do Exist?

P. L. F. **Haberler**

Institut für Hochenergiephysik der Österreich, Akademie der Weissenschaften - Wien

(ricevuto il 10 Aprile 1972)

It has been pointed out that several features of the Stanford experiments, as interpreted according to the ideas of scaling, seem to resemble the behaviour on the light-cone of free field theory or interacting field theory with naive manipulation of operators rather than the behaviour of renormalized perturbation expansions of renormalizable field theories. Thus free-field theory models, in particular the light-cone behaviour of local and bilocal operators in a free-quark theory (1), have been studied for the purpose of abstracting algebraic relations that might be true on the light-cone in the real world of hadrons. The abstraction of free-quark light-cone commutation relations becomes more credible, if one can show that certain kinds of nontrivial interactions of quarks leave the relations undisturbed, according to the method of naive manipulation of operators. There is evidence (2-5) that in fact this is so in a theory with neutral scalar or pseudoscalar « gluons » having a Yukawa interactions with the quarks. On the contrary, if the « gluon » is a vector boson additional singularities (6) seem to arise, which destroy the basic quark light-cone algebra. Therefore two questions arise (7). The first one is whether one can distinguish in a well-defined way, using physical quantities, between a theory that makes use of SU_3 triplet representations locally and one that does not. If one can, one must secondly ask whether a theory that has triplets locally necessarily implies the existence of real triplets (say real quarks) asymptotically. It is the subject of this note to suggest an answer to these questions.

First we would like to suggest a simple answer to the second question. If namely the simple light-cone algebra of Fritzsch and Gell-Mann implies (1) that the quark propagator $S^q_F(p)$ behaves at high energies like a free propagator, then using standard assumptions (8) one is led to the conclusion that quarks do exist asymptotically.

---

(1) H. Fritzsch and M. Gell-Mann: Talk presented at *International Conference on Duality and Symmetry, Tel-Aviv, Israel, April 5–7, 1971.*

(2) J. M. Cornwall and R. Jackiw: UCLA preprint (1971).

(3) D. J. Gross and S. B. Treiman: Princeton University preprint (1971).

(4) G. Domokos: Johns Hopkins University preprint NYO-4076-18 (1971).

(5) C. H. Llewellyn Smith: to be published.

(6) W. Bardeen: private communication to M. Gell-Mann; P. L. F. Haberler: to be published.

(7) R. Dashen: private communication to M. Gell-Mann.

(8) H. Lehmann, K. Symanzik and W. Zimmermann: *Nuovo Cimento*, **1**, 1425 (1955).

The key relation to show this is the asymptotic condition (⁹)

$$\psi_q(x) \to Z_q^{\frac{1}{2}} \psi_q^{in} \quad \text{as} \quad t \to -\infty\,, \tag{1}$$

which has to be understood in the sense of weak operator convergence. Z_q is the wave function renormalization constant of the quark field, which can vary between zero and one if we assume a positive metric in Hilbert space and equal-time anticommutation relations for ψ_q. To get a further estimate on Z_q, we consider the spectral representation for $S_F^q(p)$

$$S_F^q(p) = \frac{1}{\gamma p - m_q} + \int \mathrm{d}a^2 \, \frac{\varrho_1(a^2)\not{p} + \varrho_2(a^2)}{p^2 - a^2 + i\varepsilon}\,, \tag{2}$$

where all quantities are understood to be renormalized. In the limit $\not{p} \to \infty$ we obtain from (2)

$$S_F^q(p) \xrightarrow[p \gg m_q]{} \frac{1}{\gamma \cdot p}\left[1 + \int \mathrm{d}a^2 \, \varrho_1(a^2)\right] \xrightarrow[p \gg m_q]{} \frac{1}{\gamma \cdot p} Z_q^{-1}\,, \tag{3}$$

according to the Källen (¹⁰) theorem. If now $S_F^q(p)$ behaves like a free propagator at high energies,

$$S_F^q \sim c/\gamma \cdot p\,, \qquad c \text{ finite} \tag{4}$$

then, on comparing (3) and (4), we deduce

$$c = Z_q^{-1} = \text{finite}\,. \tag{5}$$

As a consequence of (5) we can conclude from (1) that *the Gell-Mann–Fritzsch light-cone algebra scheme implies that real quarks exist asymptotically.*

This rather startling result holds under the strong assumptions made, namely

a) existence of a unique vacuum,

b) canonical anticommutation relations (CAC),

c) positive metric,

d) locality.

But are these assumptions really compatible with a theory that can describe interactions? After all we are working with a mathematical idealization of a physical theory in terms of local Lagrangians and field operators and it may be indeed true that the present formalism is the *local* limit of some unknown, more elaborate theory. As we will see in the following discussion there are strong physical and mathematical indications that at least one of the above assumptions cannot hold in a nontrivial interacting theory.

(⁹) Internal symmetry indices are suppressed.

(¹⁰) G. KÄLLEN: *Helv. Phys. Acta*, **25**, 417 (1952).

First it is interesting to realize that for a dilatation invariant *interacting* theory the short-distance behaviour of the two-point function must in general be different from the free two-point function because of a well-known theorem (¹¹).

Theorem. If the two-point function is equal to that of a free zero-mass theory, the field is necessarily a free field.

This tells us that if there is going to be any connection between the dilatation group and the short-distance behaviour, the dimension of the filed should change, because otherwise the theory would be that of a free field.

Similarly the ICAR theorem (¹²) tells us:
If a local relativistic Fermi field satisfies 1) CAC, 2) irreducibility of the fields at a fixed point and 3) a certain limitation of the growth of the spectral function at infinity, which is slightly stronger than the requirement that the mass renormalizations, δM, is finite, then the Fermi field is a free field.

These theorems at least strongly suggest that it will be difficult to maintain assumptions *a*)-*d*) in an interacting field theory. Indeed, recently it became quite clear (¹³) that the uniqueness of the vacuum state has to be given up, if one wants at least approximately a free field structure for certain single-particle operators. This degeneracy of the ground state comes about because the ground state is filled with soft scalar and pseudoscalar mesons and we know from the situation found in the theory of superconductivity that a Hartree-Fock approximation, which amounts in this case of taking the ground-state expectation values of certain pair operators, will give good results because the number of these quanta is large, *i.e.* the effective interaction is strong. This leads then to the somewhat paradoxically sounding result: *the stronger the degeneracy of the vacuum state, the closer are the equations of motion of the single-particle operators to the free equations.* It can then be shown that the interaction is *explicitly* given in form of an eikonal-phase operator. This *explicit* solution of a lightlike quark model with a fully degenerate vacuum provides a culmination point in the development of current algebra which started with the famous papers by NAMBU and JONA-LASINIO (¹⁴). In this solution the hypotheses of a parton picture on the one hand and dual-resonance picture (Veneziano model) on the other hand seem to be successfully combined. Will this solution be close to reality?

First let as remark (¹⁵) that the parton picture can be understood quite well from a classical point of view. This is intuitively clear because the results of the light-cone algebra are obtained by a naive manipulation of field operators. This suggests therefore that important quantum effects might have been neglected in this formalism. Indeed, it is a well-known fact that *unitarity* corrections are neglected in the usual current-algebra approach (Veneziano model). Although this approximations may work quite well at low energies, the following important questions of

A) a unique physical vacuum;

B) the contribution of electromagnetic, weak and gravity interactions at high energies;

C) anomalous singularities, Schwinger-Adler anomalies;

(¹¹) Its nontrivial generalization to zero mass was demonstrated by K. POHLMEYER: *Comm. Math. Phys.*, **12**, 204 (1969).

(¹²) R. POWERS: *Comm. Math. Phys.*, **4**, 145 (1967). This theorem is a weaker version of a similar theorem by G. KÄLLEN: *Suppl. Acta Phys. Austr.*, **4**, 133 (1965) and references therein.

(¹³) See ref. (⁴) and references quoted therein.

(¹⁴) Y. NAMBU and G. JONA-LASINIO: *Phys. Rev.*, **122**, 345 (1961); **124**, 246 (1961).

(¹⁵) K. DIETZ: Seminar presented at the Schladming Winter School, March 1972.

D) conformal invariance and bootstrap;

E) wee partrns, relation between elastic and inelastic processes,

still remain open. We will discuss shortly these questions in turn.

A) An asymmetric vacuum, as was the case discussed above, implies that there exist an infinity of possible degenerate vacua, all physically equivalent, differing by each other only in the number of soft mesons. To choose one particular vacuum, one has to break the symmetry explicitly to obtain a unique vacuum state. This explicit symmetry breaking is nothing else than the effect of another interaction and only if we have taken all possible interactions into account, we can expect that we obtain the true physical vacuum. Now it has become clear recently that these breaking effects have important nonnegligible effects. It has been shown for example by Cabibbo (16) that, if these breaking effects are considered to be only small, then one obtains *unphysical* solutions, *e.g.* $\theta_C = 0$ or 90° for the Cabibbo angle. Therefore electrrmagnetic and weak corrections may become indeed very important. It was particularly stressed recently bu Ueda and Schechter (17) that electromagnetism plays not only an important role in selecting the physical vacuum but also in establishing the ground state which may give rise to tadpoles, a truly important effect at high energies.

B) As everybody knows electromagnetic and weak correction diverge at high energies. This may be an indication that this interactions give rise to large effects (18) or may even dominate (19) the strong interactions at high energies.

C) Since the electromagnetic interactions are transmitted by a vector gluon, namely a photon, we certainly expect that all sorts of anomalous singularities do arise, which will effect the basic quark light-cone algebra. It is interesting to remark in this context that the Schwinger-Adler anomaly seems to imply (20) parastatistics for the quarks, making it less welcome that quarks do exist (1,21). Although the Schwinger-Adler anomaly is regarded as a low-energy effect, it can be shown (22) that it is actually a *high*-energy effect, which has *also* an effect on low-energy phenomena. An indication for such a situation lies in the fact, that the anomaly is given exactly by the unrenormalized, *i.e.* bare, form of the axial vector divergence. Furthermore, it has been shown by Wess and Zumino in a beautiful paper (23) that there are strong indications that these anomalies are really needed to explain experimental results. It is therefore very unlikely that these anomalies cancel out internally in a more fundamental theory. Concerning anomalies we would once more draw attention to the fact that current algebra has *not yet* really succeeded to explain the $\eta \to 3\pi$ decay, the η' decay which bears on the question whether there exists an *operator term* δ that breaks scale invariance, the K_{l_3} decay and the $\Delta I = 1$ electromagnetic mass differences. Furthermore, also in these cases the electromagnetic breaking (17) of the basic $SU_3 \times SU_3$ symmetry seems to be of great importance to understand the experimental facts.

---

(16) N. Cabibbo: *Hadrons and Their Interactions*, edited by A. Zichichi (New York, 1968).

(17) J. Schechter and Y. Ueda: *Phys. Rev. D*, **4**, 742 (1971) and references therein.

(18) See, *e.g.*, F. E. Low and S. B. Treiman: *Phys. Rev. D*, to be published.

(19) G. Berlad, A. Dar, G. Eilam and J. Franklin: Technion preprint, 1971.

(20) See the discussion in Sect. **9** of ref. (1).

(21) M. Gell-Mann: *Lectures Given at the Schladming Winter School* (March 1972).

(22) P. L. F. Haberler: to be published.

(23) J. Wess and B. Zumino: *Phys. Lett.*, **37** B, 95 (1971); J. Wess: *Lectures Given at the Schladming Winter School* (March 1972).

D) Recent study of the implications of scale and conformal invariance strongly suggests that conformal invariance seems to be closely related to the bootstrap (24). In this context it is interesting to note that *anomalous* dimensions seems to be necessary to obtain an ultraviolet—and infrared—divergence free conformal invariant theory. Canonical dimensions on the contrary seem to lead to divergence difficulties or the existence of an *infinite* number of conserved tensors (25). Indeed, it was already observed in ref. (1) that in the case that the Gell-Man–Low function vanishes (26) for a special value of the coupling constant the asymptotic behaviour of the vacuum expectation value of the current products or current commutators has a different (*reduced*) singularity as compared to naive or free-quark behaviour. This strongly suggests that assumptions *b*) and probably also (27) *d*) cannot be maintained in a conformal invariant *interacting* theory.

E) Finally we would like to mention the problem of the so-called wee partons (28) which are connected with the production of *soft* particles in high-energy collisions. The wee partons do *break* scale invariance, never the less they may lead to an understanding (29) of the observed limited transverse cut-off of secondaries produced in high-energy collisions. There are strong indications that these «wee partons» have to be there since otherwise unitarity is violated.

To sum up. From a theoretical point of view we learn from the discussion of questions *A*)-*E*)—which seem to be closely connected—that the quark light-cone algebra of Fritzsch and Gell-Mann can only hold in a *free*-quark model if it implies that the quark propagator acts free at high energies. This would mean that the existence of free quarks is implicitly postulated in this framework.

From an experimental point of view we predict various violations of the quark light-cone algebra, connected with the existence of vector gluons and the breakdown of scale invariance at very high energies.

* * *

Useful discussions with Prof. M. Gell-Mann are greatfully acknowledged.

---

(24) A. M. Polyakov: *Sov. Phys. JETP Lett.*, **12**, 538 (1970); P. L. F. Haberler: *Proceedings of the Symposium on Conformal and de Sitter Groups, June 1970* (Boulder, 1971); A. A. Migdal: *Phys. Lett.*, **37** B, 98, 386 (1971); P. L. F. Haberler: *Phys. Lett.*, **34** B, 75 (1971); **37** B, 71 (1971); and MPI preprint October 1971, to be published; G. Mack and I. Todorov: Trieste preprint IC/71/139.

(25) See in this context also the interesting paper by S. Ferrara, A. F. Grillo, G. Parisi and R. Gatto: *Phys. Lett.*, **38** B, 333 (1972).

(26) Under certain conditions the vanishing of the Gell-Mann low function can imply conformal invariance, see *e.g.*, B. Schroer: *Lett. Nuovo Cimento*, **2**, 867 (1971).

(27) L. Castell: private communication; see also the fifth paper in ref. (24).

(28) S. D. Drell and T. M. Yan: *Ann. of Phys.*, **66**, 578 (1971).

(29) L. Stodolsky: SLAC-PUB-864, March (1971), unpublished.