CANONICAL DIMENSIONS FOR THE ELECTROMAGNETIC FIELD AND CURRENT IN QUANTUM ELECTRODYNAMICS

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It is suggested that the Gell-Mann Low function vanishes for a fixed (large) value of the unrenormalized coupling constant.

Recently the question whether scale invariance can be regarded as a good asymptotic symmetry has received considerable attention in literature [1-8]. In various field theoretic models and in perturbation theory anomalous dimensions [1] and intrinsic breaking of scale invariance were reported [1-5]. In particular, it was remarked that also in quantum electrodynamics (QED) scale invariance appears to be intrinsically broken [2], at least in perturbation theory. Therefore, the concept of dimension, even for currents, seems to be lost.

On the other hand, the MIT-SLAC electroproduction experiments [9], which gave the initial impetus to the study of scale symmetry, provide no support for any anomalous behaviour of the electromagnetic current [10,11].

Since there seems to be some indication [9] that perturbation theory does not describe the experimental situation satisfactorily one can ask oneself whether a non-perturbative approach to QED leads to a better situation. Now the only non-perturbative method available is the renormalization group [12] approach. Recently considerable progress [13] has been made in this field of research. In particular Jouvet and Astaud [13] have clearly stated the necessary conditions which have to be fulfilled that the Gell-Mann Low [14] function $\Psi(\alpha_0)$ vanishes for a fixed value of the unrenormalized coupling constant α_0. Due to the work of Callan [2], Symanzik [3] and others $\Psi(\alpha_0^{fix}) = 0$ in turn implies that scale invariance is a good asymptotic symmetry of the theory.

In this communication we present a summary of our recent investigations which suggest that $\Psi(\alpha_0^{fix})$ indeed vanishes in QED. The details of our investigations are deferred to an extended publication.

The results are as follows:

Although due to Adler anomalies in the trace of the energy-momentum tensor the dimensions d_{A_μ} and d_{j_μ} of the electromagnetic field and current respectively are anomalous in lowest order

$$d_{A_\mu} = 1 + \alpha/3\pi$$
$$d_{j_\mu} = 3 + \alpha/3\pi \qquad (1)$$

($\alpha = e^2/4\pi$, e is the renormalized coupling constant) and undefined in higher orders of perturbation theory, a global solution of QED has the property that d_{A_μ} and d_{j_μ} have canonical dimensions.

This result follows from the fact that the Gell-Mann Low function vanishes for a fixed value of α_0

$$\Psi(\alpha_0 = \alpha_0^{fix}) = 0 \qquad (2)$$

and therefore the term which is responsible for the intrinsic breaking of scale invariance drops out in the Callan-Symanzik [2,3] equations and scale invariance becomes a good asymptotic symmetry in QED. Eq. (2) is implied by a positive energy spectrum and by a bootstrap condition* for the Feinstructure constant

$$Z_2(\alpha) = 0 \qquad (3)$$

which in turn leads to the result that physical masses are totally of dynamical origin. The bootstrap condition (eq. (3)) together with an explicit estimate of α_0^{fix} (see below) and the condition $\alpha_0^{fix} > \alpha$ suggest that α_0^{fix} can be associated with a strong interaction coupling constant and that the solutions of the field equations for QED include those of a strong interaction theory. This

* The bootstrap condition was already proposed in ref. [15].

would mean that pure QED is valid only below a certain cutoff momentum.

Contrary to the canonical dimensions of the electromagnetic current and field, which essentially follow from gauge invariance and a finite Z_3, the dimension of the fermion field is expected to be anomalous due to eq. (3). This means that the interacting theory is more singular than the free theory or in other words that strict locality is lost because the fermion shows some structure. Furthermore we propose now that the deviations from the canonical dimension of the fermion field at very high energies are a measure of higher [16] spin contributions. This interpretation is not altogether implausible because a connection between integer "spin" s and dimension d was noted before by Mack [16] and others, namely

$$d = s + 2$$

Let us now sketch the derivation of our results* To facilitate a better understanding we summarize first the necessary conditions for the vanishing of $\Psi(\alpha_0)$.

Define the renormalized gauge invariant photon propagator function $d_R(k^2, \alpha)$ by

$$D_R(k^2) = -1/k^2\, d_R(k^2, \alpha) \tag{4}$$

then using

$$\alpha = \alpha_0 Z_3, \qquad D_R = Z_3^{-1} D_0 \tag{5}$$

where D_0 is the unrenormalized photon propagator, one easily establishes the functional equation of the renormalization group. From the differential equation one gets the following exact integral representation for $d_R(k^2/M^2, \alpha)$, (M is the physical mass of the fermion)

$$d_R(k^2/M^2, \alpha) = 1 - \alpha \int_0^{k^2/M^2} \frac{dt}{t} H(\alpha, \alpha/d_R(t,\alpha)) \tag{6}$$

where $H(\alpha, \beta)$ is unknown. The more familiar representation of $d_R(k^2/M^2, \alpha)$ in terms of $\Psi(\beta)$ is obtained by taking M or M_0 (bare fermion mass) equal to zero

$$d_R^{M=0}(k^2/M^2, \alpha) = 1 - \alpha \int_0^{k^2/M^2} \frac{dt}{t} \Psi\left(\frac{1}{t}, \frac{\alpha}{d_R(t, \alpha)}\right), \quad M \to 0 \tag{7}$$

and

$$d_R^{M_0=0}(k^2/M_0^2, \alpha) = 1 - \alpha \int_0^{k^2/M_0^2} \frac{dt}{t} \Psi\left(\frac{1}{t}, \frac{\alpha}{d_R(t, \alpha)}\right), \quad \text{but } M \neq 0 \tag{8}$$

It is the latter representation eq. (8) which will be interesting for us in the following. Before discussing $d_R^{M_0=0}$ further we would like to restrict the general, unknown function $H(\alpha, \beta)$ by imposing the following physical desirable conditions to have a connection with QED:

1. Regularity of d_R at $k^2 = 0$, i.e. $H(\alpha, \beta)$ has an expansion around $\beta = \alpha$, such that $H(\alpha, \alpha) = 0$, $\alpha^2 \partial H(\alpha, \beta)/\partial\beta|_{\alpha=\beta} = -1$.

2. Unitary (positivity) requirement for d_R, which restricts [17,18] $Z_3 = \lim_{k^2 \to \infty} d_R(k^2/M^2, \alpha)$ in the well known way to

$$0 \leqslant Z_3 < 1 \tag{9}$$

and leads to $H(\alpha, \beta) > 0$ for $0 < \beta < \alpha_0$ with $H(\alpha, \beta_0 = \alpha \,|\, Z_3) = 0$.

3. The existence of a lowest bound state in d_R, namely the positronium, which implies the following form of $H(\alpha, \beta)$ near the pole:

$$H(\alpha, \beta)\,|_{\beta \to 0} \to -4/\beta^2\alpha^2 \tag{10}$$

There is then no contradiction with the renormalization group to have a unique solution

$$\alpha_0 = \alpha_0(\alpha) \quad \text{of} \quad H(\alpha, \alpha_0) = 0, \ \alpha_0 > \alpha \tag{11}$$

whereby α_0 is such that

$$\partial(\alpha_0(\alpha))\partial\alpha > 0 \tag{12}$$

which is the basic [19] content of the renormalization group †.

Now we are in the position of discussing eq. (8) further. Making the change of variables $t = t'/M_0^2$, which leads to

$$d_R^{M_0=0}(k^2/M_0^2, \alpha) = 1 - \alpha \int_0^{k^2} \frac{dt'}{t'} \Psi\left(\frac{M_0^2}{t'} \frac{\alpha}{d_R(t'/M_0^2, \alpha)}\right) \tag{13}$$

and then letting $M_0 \to 0$ and using $\alpha/d_R(\infty, \alpha) = \alpha_0$, we obtain

* We closely follow here the work of Astaud and Jouvet [13].

† Wilson [20] has a slightly different interpretation of the physical content of the renormalization group, namely that α_0 is independent of α. However, this invalidates Dyson's way of renormalization. Only in exceptional cases such a situation is expected to prevail. Furthermore, on our discussion we do not put M equal to zero.

$$d_R^{M_0=0}(k^2/M_0^2,\alpha) = 1 - \Psi(0,\alpha_0)\alpha \int_0^{k^2} \frac{dt'}{t'} \tag{14}$$

where $\Psi(0,\alpha_0) = \Psi(\alpha_0)$. Since the integral eq. (14) diverges which would be in contradiction to eq. (9), we conclude that $\Psi(\alpha_0)$ has to vanish for $\alpha_0 = \alpha_0^{\text{fix}}$. There is then no contradiction to having α_0 fixed since α is also fixed by

$$M_0(\alpha_{M_0=0}) = 0 \tag{15}$$

in conformity with eq. (12). Furthermore, we observe that because of eq. (10) $H(\alpha_{M_0=0},\beta)$ has no Taylor expansion but only a limited but only a limited Laurent expansion around $\alpha_0 = 0$. In other words, the usual perturbation expension is an invalid expansion, and must lead to troubles as first remarked by Dyson [21].

Having now stated the necessary conditions for the vanishing of $\Psi(\alpha_0)$ we are still left to show that M_0 actually vanishes in QED. In a previous communication strong arguments for the following relation between M and M_0 were given [15]

$$M_0 = MZ_2 \tag{16}$$

where Z_2 is the wave function renormalization constant of the fermion. This relation was derived in the Fried-Yennie gauge and agrees exactly in lowest order with the result obtained in the Coulomb gauge both evaluated in the same Lorentz frame (restframe of the particle). Furthermore eq. (16) holds also in Schwinger's covariant radiation gauge [22] not only in lowest order but also in general* using a result of Jackiw and Soloviev [23]. Since eq. (16) holds in different gauges, it must be a gauge independent result. This is confirmed by a recent work of Haller and Landovitz [24], who have shown that Z_2 can be defined in a gauge independent way if the subsidiary condition (Lorentz condition) is properly taken into account for interacting photons and electrons. Specifically they were able to show that Z_2, when calculated in one of the usual covariant gauges but with interacting subsidiary condition as a constraint, reduces exactly to the expression obtained in the radiation gauge, which is independent of these modofications because it works only with the physical transverse photon states. Since eq. (16) holds in the radiation gauge, we conclude therefore that it holds in all gauges.

* In this context we want to stress the fact that the assumption of positive energy spectrum is essential for the validity of eq. (16) in QED.

But now it is not difficult any more to show that M_0 vanishes. Källén [25] has given a proof that Z_2 vanishes in QED. This result has been obtained in a special gauge, however one can show that it holds in all gauges and the argument goes as follows [26-28]:

Källén's formulation of QED is based on one of the true [27] gauges where the asymptotic condition can be formulated for the electromagnetic field. Furthermore the renormalized fermion field $\psi(x)$, from which Z_2 is defined through the equal time anticommutator

$$\{\psi(x), \bar\psi(y)\}_{x_0=y_0} = Z_2^{-1}\gamma^0\delta^3(x-y) \tag{17}$$

is related to the fermion field $\psi^{T}(x)$ in the true transverse gauge by an operator gauge transformation.

$$\psi(x) = \exp(ie\Lambda(x))\,\psi^{T}(x)$$

where (18)

$$\Lambda(x) = (\partial^2)^{-1}\partial_\mu A^\mu(x)$$

$\Lambda(x)$ represents the longitudinal components of $A_\mu(x)$, i.e. $\partial_\mu\Lambda(x) = A^L_\mu(x)$. Since $\psi^T(x)$ commutes [26,27] with $\Lambda(x)$ - $\psi^T(x)$ depends only on the transverse components of $A_\mu(x)$, all the unphysical photon contributions are projected out by the Lorentz condition - we obtain

$$\{\psi(x), \bar\psi(y)\}_{x_0=y_0} = \{\psi^T(x), \bar\psi^T(y)\}_{x_0=y_0}$$

and therefore

$$Z_2^{-1} = (Z_2^{-1})^{T} \tag{19}$$

If we therefore accept that Z_2 vanishes in all gauges, then M_0 also vanishes due to eq. (16) and the necessary conditions for the vanishing of $\Psi(\alpha_0)$ seem to be fulfilled. At this point we have to be somewhat careful, however, because Källén interpreted his result in that way that QED for itself cannot be fully consistent or in other words that Z_2 is trivially zero [29]. To avoid this somewhat negative conclusion, which is not at all confirmed by experiment [30], we assume that the theory is cut off in some way - for example by gravitation [31], as suggested a long time ago by Weisskopf - and interpret then Källén's result as an eigenvalue equation for α in terms of this cutoff. Now we can come back to an estimate of α_0^{fix}, using Padé summation methods [32]:

$$\alpha_0^{\text{fix}}/2\pi \approx 1.111$$

or

$$\alpha_0^{\text{fix}} \approx 7 \tag{20}$$

which is just one half of the value of the usual pion-nucleon coupling constant. This interpretation suggests itself due to the following facts: Since the photon has two degrees of freedom as compared to one of a single pion the factor one half is intriguing. This factor one half is also borne out in a comparison of a perturbation theoretic calculation of the spectral function of the fermion propagator in QED and pseudoscalar pion-nucleon theory with massless pion, respectively. This therefore strongly suggests that α_0^{fix} is a strong interaction coupling constant. Furthermore it was already suggested before [33] that the field equations for QED may also have a strong coupling solution, the conservation of baryonic charge being then nothing else than an aspect of gauge invariance. As a final support of our argumentation - which is based mainly on the nonperturbative method of the renormalization group - we would like to mention the remarkable mass formula of Nambu [34], which relates the masses of all particles to the mass of the electron and the finestructure constant. This opens the possibility that all interactions are related to the electromagnetic interaction and ultimately to the gravitational interaction.

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