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Breakdown of Scaling Invariance in the Generalized Bjorken Limit of Perturbation Theory (*).

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Summary. — We examine the cross-sections for meson electroproduction in two model field theories. We find that for the interaction of neutral scalar mesons with nucleons the structure functions exhibit scaling in the *generalized* Bjorken limit, but that for the case of the charge-independent pseudoscalar-meson-nucleon interaction scaling does not always occur. We comment on the origin of this breakdown and on its significance for parton models of deeply inelastic electron-hadron scattering.

1. – Introduction.

The recent suggestion of BJORKEN (1) that the cross-section for inelastic electron-hadron scattering processes in which only the final electron is observed should depend only on dimensionless ratios of the kinematic invariants of the scattering process has been reasonably successful in describing the data available for such processes (2). This success has led several authors (3,4) to speculate that similar behaviour, generally termed « scaling », should also occur for « semi-inclusive » processes, *i.e.* those in which both the final electron and a single final-state hadron are detected. Our purpose in this note is to examine whether or not such a hypothesis is supported by detailed calculations based on field

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(1) J. D. BJORKEN: *Phys. Rev.*, **179**, 1547 (1969).
(2) J. D. BJORKEN and E. A. PASCHOS: *Phys. Rev.*, **185**, 1975 (1969).
(3) R. F. KÖGERLER and R. M. MURADYAN: Dubna preprint E2-4791 (unpublished).
(4) S. D. DRELL and TUNG-MOW YAN: *Phys. Rev. Lett.*, **24**, 855 (1970).

theory in lowest-order perturbation approximation. While conclusions based on such a simple model cannot be regarded as definitive, they do suggest that a careful re-examination of the bases for this hypothesis is in order.

2. – Kinematic analysis of the semi-inclusive cross-section.

The general process which we wish to examine is shown in Fig. 1 *a*), and it is easy to show that the differential cross-section for the process can be written in the form

$$\mathrm{d}\sigma = \frac{1}{4\sqrt{pk - M^2 M_e^2}}\,\frac{e^4}{q^4}\,\frac{1}{(2\pi)^6}\,\frac{\mathrm{d}^3k'}{2k_0'}\,\frac{\mathrm{d}^3p'}{2p_0'}\,\varrho_{\mu\nu}\,j^{\mu\nu}\,, \tag{1}$$

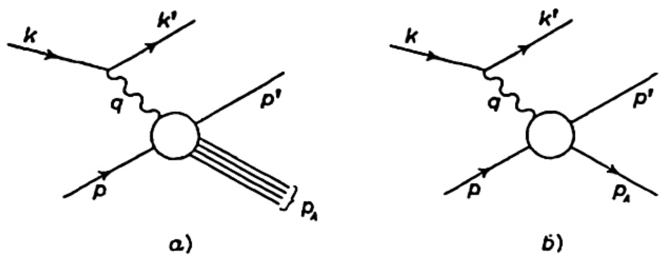

Fig. 1. – Kinematics for electroproduction of a meson of momentum p' from a proton: *a*) general case; *b*) case in which the unobserved hadron state consists of a single nucleon.

where M is the mass of the target, M_e is the electron mass,

$$j_{\mu\nu} = \frac{1}{2}\,\mathrm{Tr}\left\{\gamma_\mu((\gamma\cdot k) + M_e)\gamma_\nu((\gamma\cdot k') + M_e)\,\frac{(1+\gamma_5(\gamma\cdot s'))}{2}\right\} \tag{2}$$

and

$$\varrho_{\mu\nu} = \sum_A (2\pi)^4\,\delta^{(4)}(p+q-p'-p_A)\langle p|J_\mu|p'p_A\rangle\langle p'p_A|J_\nu|p\rangle \tag{3}$$

with J_μ the hadronic electromagnetic current.

The requirements of gauge invariance and parity invariance guarantee that the hadronic current tensor $\varrho_{\mu\nu}$ can be written in the form

$$\varrho_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)\cdot \tag{4}$$

$$\cdot\,\varrho_1 + \mathsf{P}_\mu\mathsf{P}_\nu\varrho_2 + \mathsf{P}'_\mu\mathsf{P}'_\nu\varrho_3 + (\mathsf{P}_\mu\mathsf{P}'_\nu + \mathsf{P}_\nu\mathsf{P}'_\mu)\varrho_4 + (\mathsf{P}_\mu\mathsf{P}'_\nu - \mathsf{P}_\nu\mathsf{P}'_\mu)\varrho_5\,,$$

where, following KÖGERLER and MURADYAN ([3]), we have defined

$$\mathrm{P}^{\mu} = p^{\mu} - \frac{p\cdot q}{q^2}\, q^{\mu}\,, \tag{5a}$$

$$\mathrm{P}^{\mu'} = p^{\mu'} - \frac{p'\cdot q}{q^2}\, q^{\mu}\,. \tag{5b}$$

3. – Structure functions for meson electroproduction.

We shall now compute the structure functions ϱ_1, ϱ_2, ϱ_3, ϱ_4 and ϱ_5 for the special case in which the observed hadron p' is a meson and the unobserved hadron p_A is a single nucleon as indicated in Fig. 1 *b*). We consider the cases in which:

a) The observed meson is a neutral scalar meson σ of mass m interacting with the nucleons through the Hamiltonian

$$H_{\sigma} = g_0\, \bar{\psi}\psi\sigma\,. \tag{6}$$

The lowest-order Feynman diagrams which contribute to this process are presented in Fig. 2 *a*).

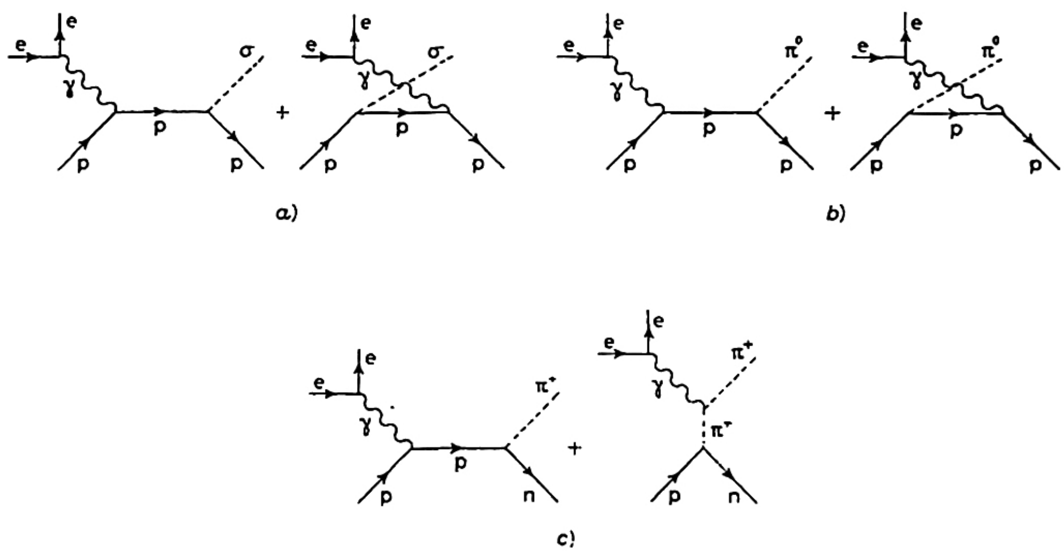

Fig. 2. – Lowest-order Feynman diagrams for the process of Fig. 1*b*) for *a*) σ production; *b*) π^0 production; *c*) π^+ production.

b) The observed meson is a pion $\boldsymbol{\varphi}$ of mass m interacting with the nucleons through the Hamiltonian

$$H_{\pi} = ig\bar{\psi}\gamma_5\boldsymbol{\tau}\psi\cdot\boldsymbol{\varphi}\,. \tag{7}$$

The lowest-order diagrams contributing for the case of a neutral pion are shown in Fig. 2 *b*), and those which contribute for the case of a positively charged pion are shown in Fig. 2 *c*).

Straightforward calculation yields the following expressions for the structure functions:

1) σ case:

$$\varrho_1 = \frac{4\pi g_0^2 (p'\cdot q)^2[2p\cdot p'(q^2+2p\cdot q) - 2m^2 p\cdot q - 4M^2q^2]}{(q^2+2p\cdot q)^2(m^2-2p'\cdot p)^2}\,\delta((p+q-p')^2-M^2)\,, \tag{8a}$$

$$\varrho_2 = \frac{16\pi g_0^2(p'\cdot q)^2(m^2-4M^2)}{(q^2+2p\cdot q)^2(m^2-2p\cdot p')^2}\,\delta((p+q-p')^2-M^2)\,, \tag{8b}$$

$$\varrho_3 = \frac{4\pi g_0^2[(4M^2-2p\cdot p')(q^2+2p\cdot q)-2p\cdot q(m^2-2p\cdot p')]}{(q^2+2p\cdot q)(m^2-2p\cdot p')^2}\cdot \delta((p+q-p')^2-M^2)\,, \tag{8c}$$

$$\varrho_4 = \frac{8\pi g_0^2(q\cdot p')(m^2-4M^2)}{(q^2+2p\cdot q)(m^2-2p'\cdot p)^2}\,\delta((p+q-p')^2-M^2)\,, \tag{8d}$$

$$\varrho_5 = 0\,. \tag{8e}$$

2) π^0 case:

$$\varrho_1 = \frac{8\pi g^2(p'q)^2[2p\cdot p'(p'p+p'q)-m^2(p'p+pq)]}{(q^2+2pq)^2(m^2-2p\cdot p')^2}\,\delta((p+q\cdot p')^2-M^2)\,, \tag{9a}$$

$$\varrho_2 = \frac{-16\pi g^2 m^2(p'\cdot q)^2}{(q^2+2p\cdot q)^2(m^2-2p\cdot p')^2}\,\delta((p+q-p')^2-M^2)\,, \tag{9b}$$

$$\varrho_3 = \frac{-4\pi g^2[2q^2p\cdot p' + 2m^2(p\cdot q)]}{(q^2+2p\cdot q)(m^2-2p\cdot p')^2}\,\delta((p+q-p')^2-M^2)\,, \tag{9c}$$

$$\varrho_4 = \frac{8\pi g^2 m^2(p'\cdot q)}{(q^2+2p\cdot q)(m^2-2p'\cdot p)^2}\,\delta((p+q-p')^2-M^2)\,, \tag{9d}$$

$$\varrho_5 = 0\,. \tag{9e}$$

3) π^+ case:

$$\varrho_1 = \frac{8\pi g^2[4p'p(p'p+p'q)-2m^2[p\cdot p'+p\cdot q]]}{2(q^2+2pq)^2}\,\delta((p+q-p')^2-M^2)\,, \tag{10a}$$

$$\varrho_2 = \frac{-16\pi g^2 m^2}{(q^2+2p\cdot q)^2}\,\delta((p+q-p')^2-M^2)\,, \tag{10b}$$

$$\varrho_3 = \frac{-4\pi g^2[4q^2(q^2-2p'q)+4m^2(q^2+2p\cdot q)]}{(q^2+2p\cdot q)(q^2-2p'q)^2}\,\delta((p+q-p')^2-M^2)\,, \tag{10c}$$

$$\varrho_4 = \frac{-4\pi g^2(4M^2-2q^2)}{(q^2-2p'q)(q^2+2pq)}\,\delta((p+q-p')^2-M^2)\,, \tag{10d}$$

$$\varrho_5 = 0\,. \tag{10e}$$

The structure functions ϱ_i depend on the invariants

$$Q^2 \equiv -q^2\,, \qquad \nu \equiv p\cdot q\,, \qquad \nu_1 = p'\cdot q\,, \qquad \varkappa \equiv p'\cdot p \tag{11}$$

and have the dimensions

$$\begin{cases} [\varrho_1] = \text{mass}^{-2}\,, \\ [\varrho_a] = \text{mass}^{-4}\,, & a = 2, \ldots, 5\,. \end{cases} \tag{12}$$

If invariance under the scale transformations

$$\begin{cases} q \to \lambda q\,, \\ p \to \lambda p\,, \\ p' \to \lambda p'\,, \end{cases} \tag{13}$$

is to hold, it is necessary that the structure functions satisfy the equations [5]

$$\lambda^2 \varrho_1(\lambda^2 Q^2, \lambda^2 p\cdot q, \lambda^2 p'\cdot q, \lambda^2 p'\cdot p) = \varrho_1(Q^2, p\cdot q, p'\cdot q, p'\cdot p)\,, \tag{14a}$$

$$\lambda^4 \varrho_a(\lambda^2 Q^2, \lambda^2 p\cdot q, \lambda^2 p'\cdot q, \lambda^2 p'\cdot p) = \varrho_a(Q^2, p\cdot q, p'\cdot q, p'\cdot p)\,, \qquad a = 2, \ldots, 5\,. \tag{14b}$$

We now pass to the generalized Bjorken limit

$$\begin{cases} \nu,\ Q^2,\ -\nu_1,\ \varkappa \to \infty\,, \\ \dfrac{2\nu}{Q^2} = -\dfrac{\varkappa}{\nu_1}\ \text{fixed}\,, \qquad \dfrac{\nu_1}{\nu}\ \text{fixed}\,, \end{cases} \tag{15}$$

define

$$\omega \equiv \frac{2\nu}{Q^2}\,, \tag{16a}$$

$$\alpha \equiv \frac{\nu_1}{\nu}\,, \tag{16b}$$

and observe that the scaling requirements of eqs. (14) are met in this limit if and only if

$$\lim_{\text{Gen. Bj.}} \nu \varrho_1 = F_1(\omega, \alpha)\,, \tag{17a}$$

$$\lim_{\text{Gen. Bj.}} \nu^2 \varrho_a = F_a(\omega, \alpha)\,, \qquad a = 2, \ldots, 5\,, \tag{17b}$$

with all the F_i dimensionless functions of ω and α. We now evaluate the left-

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(5) V. A. MATVEEV, R. M. MURADYAN and A. N. TAVKHELIDZE: Dubna preprint E2-4698.

hand side of eqs. (17) using the structure functions we have already calculated. We find for both the π^0 and σ cases that

$$\lim_{\text{Gen. Bj.}} \nu\varrho_1 = \frac{\pi g^2}{2}\,\delta((1-\omega)(1+\alpha\omega))\,\frac{1}{\omega-1}\,, \tag{18a}$$

$$\lim_{\text{Gen. Bj.}} \nu^2\varrho_3 = \pi g^2\delta((1-\omega)(1+\alpha\omega))\,\frac{\omega}{\omega-1}\,, \tag{18b}$$

$$\lim_{\text{Gen. Bj.}} \nu^2\varrho_k = 0\,, \qquad k = 2, 4, 5\,, \tag{18c}$$

while for π^+ [6] that

$$\lim_{\text{Gen. Bj.}} \nu\varrho_1 = 2\pi g^2\,\frac{\omega^2}{\omega-1}\,\delta\left((1-\omega)(1+\alpha\omega)-\frac{m^2}{Q^2}\right), \tag{19a}$$

$$\lim_{\text{Gen. Bj.}} \nu^2\varrho_k = 0\,, \qquad k = 2, 5\,, \tag{19b}$$

$$\lim_{\text{Gen. Bj.}} \nu^2\varrho_3 = -8\pi g^2\left(\frac{\nu}{m^2}\right)\omega(\omega^2-2\omega)\,\delta\left((1-\omega)(1+\alpha\omega)-\frac{m^2}{Q^2}\right), \tag{19c}$$

$$\lim_{\text{Gen. Bj.}} \nu^2\varrho_4 = -4\pi g^2\left(\frac{\nu}{m^2}\right)\omega\delta\left((1-\omega)(1+\alpha\omega)-\frac{m^2}{Q^2}\right). \tag{19d}$$

Thus, we see that while all the structure functions for σ and π^0 production manifest the scaling behavior described in eqs. (17), the structure functions ϱ_3 and ϱ_4 for π^+ production do not.

At this point, the objection might be raised that this loss of scaling behavior is due to a particularly injudicious choice of structure functions [7]. In order to eliminate this possibility, we have also examined the generalized Bjorken limit of the structure functions for meson electroproduction defined in a manner completely at variance with eq. (4).

Following KÖGERLER and MURADYAN [3], we define the polarization vectors

$$\varepsilon_\mu^{(L)} \equiv (-P^2)^{-\frac{1}{2}} P_\mu\,, \tag{20a}$$

$$\varepsilon_\mu^{(T_1)} \equiv \left(\frac{P^2}{(P\cdot P')^2 - P^2P'^2}\right)^{\frac{1}{2}}\left(P'_\mu - \frac{P\cdot P'}{P^2}\,P_\mu\right), \tag{20b}$$

$$\varepsilon_\mu^{(T_2)} \equiv \left(q^2[(p\cdot p')^2 - M^2m^2]\right)^{-\frac{1}{2}}\varepsilon_{\mu\nu\lambda\varrho}\,p^\nu p'^\lambda p^\varrho\,, \tag{20c}$$

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[6] The factor m^2/Q^2 is retained in the delta-functions in eq. (20), so that the large Q^2 dependence of F_i may be computed.

[7] The structure functions ϱ_i, $i = 1, \ldots, 5$, are particularly convenient because they are free from kinematical singularities.

which satisfy the conditions

(21a) $$\varepsilon^{\alpha}_{\mu} q_{\mu} = 0\,, \qquad \alpha = L,\, T_1,\, T_2\,,$$

(21b) $$\sum_{\alpha=L,T_1,T_2} \varepsilon^{\alpha}_{\mu}\varepsilon^{\alpha}_{\nu} = -g_{\mu\nu} + q_{\mu}q_{\nu}/q^2\,.$$

It then follows that $\varrho_{\mu\nu}$ can be written in the form

(22) $$\varrho_{\mu\nu} = \varrho_{T_1}\varepsilon^{(T_1)}_{\mu}\varepsilon^{(T_1)}_{\nu} + \varrho_{T_2}\varepsilon^{(T_2)}_{\mu}\varepsilon^{(T_2)}_{\nu} + \varrho_L\varepsilon^{(L)}_{\mu}\varepsilon^{(L)}_{\nu} + \varrho_{TL^{(+)}}\big(\varepsilon^{(T_1)}_{\mu}\varepsilon^{(L)}_{\nu} + \varepsilon^{(L)}_{\mu}\varepsilon^{(T_1)}_{\nu}\big) + i\varrho_{TL^{(-)}}\big(\varepsilon^{(T_1)}_{\mu}\varepsilon^{(L)}_{\nu} - \varepsilon^{(L)}_{\mu}\varepsilon^{(T_1)}_{\nu}\big)\,,$$

where

(23a) $$\varrho_1 = \varrho_{T_1}\,,$$

(23b) $$\varrho_2 = \mathbf{P}^{-2}\left[\varrho_{T_1} - \varrho_L - \frac{(\mathbf{P}\cdot\mathbf{P}')^2}{\mathbf{P}^2\mathbf{P}'^2 - (\mathbf{P}\cdot\mathbf{P}')^2}(\varrho_{T_1} - \varrho_{T_2}) - \frac{2\mathbf{P}\mathbf{P}'}{[\mathbf{P}^2\mathbf{P}'^2 - (\mathbf{P}\cdot\mathbf{P}')^2]^{\frac{1}{2}}}\varrho_{TL^{(+)}}\right],$$

(23c) $$\varrho_3 = \frac{\mathbf{P}^2}{(\mathbf{P}\cdot\mathbf{P}')^2 - \mathbf{P}^2\mathbf{P}'^2}[\varrho_{T_1} - \varrho_{T_2}]\,,$$

(23d) $$\varrho_4 = \frac{1}{[\mathbf{P}^2\mathbf{P}'^2 - (\mathbf{P}\cdot\mathbf{P}')^2]^{\frac{1}{2}}}\varrho_{TL^{(+)}} + \frac{\mathbf{P}\cdot\mathbf{P}'}{\mathbf{P}^2\mathbf{P}'^2 - (\mathbf{P}\cdot\mathbf{P}')^2}(\varrho_{T_1} - \varrho_{T_2})\,,$$

(23e) $$\varrho_5 = \frac{1}{[\mathbf{P}^2\mathbf{P}'^2 - (\mathbf{P}\cdot\mathbf{P}')^2]^{\frac{1}{2}}}\varrho_{TL^{(-)}}\,.$$

The requirement of scaling for these new structure functions in the generalized Bjorken limit is simply [3,5]

(24) $$\lim_{\text{Gen. Bj.}} \varrho_a = \frac{1}{Q^2}F_a(\omega, \alpha)\,, \qquad a = T_1,\, T_2,\, L,\, TL^{(\pm)}$$

and by explicit calculation, we find for both σ and π^0 production that

(25a) $$\lim_{\text{Gen. Bj.}} \varrho_{T_1} = \frac{1}{Q^2}\pi g^2\frac{1}{\omega(\omega-1)}\delta\big((1-\omega)(1+\alpha\omega)\big)\,,$$

(25b) $$\lim_{\text{Gen. Bj.}} \varrho_{T_2} = \frac{1}{Q^2}\pi g^2\frac{1}{\omega(\omega-1)}\delta\big((1-\omega)(1+\alpha\omega)\big)\,,$$

(25c) $$\lim_{\text{Gen. Bj.}} \varrho_i = 0\,, \qquad i = L,\, TL^{(\pm)}\,,$$

while for π^+ production we have [6]

(26a) $$\lim_{\text{Gen. Bj.}} \varrho_{T_1} = \frac{4\pi g^2}{Q^2}\left[\frac{\omega}{\omega-1} + 4\left(\frac{M^2}{m^2} + \omega^2\right)\left(1 - \left(\frac{\omega}{2}\right)^{-1}\right)\right]\cdot \delta\left((1-\omega)(1+\alpha\omega) - \frac{m^2}{Q^2}\right),$$

(26b) $$\lim_{\text{Gen. Bj.}} \varrho_{T_2} = \frac{4\pi g^2}{Q^2}\,\frac{\omega}{\omega-1}\,\delta\left((1-\omega)(1+\alpha\omega)-\frac{m^2}{Q^2}\right),$$

(26c) $$\lim_{\text{Gen. Bj.}} \varrho_L = \frac{4\pi g^2}{Q^2}\,\omega(\omega-1)\,\frac{Q^2}{m^2}\,\delta\left((1-\omega)(1+\alpha\omega)-\frac{m^2}{Q^2}\right),$$

(26d) $$\lim_{\text{Gen. Bj.}} \varrho_{TL^{(+)}} = -\frac{4\pi g^2}{Q^2}(2\omega-3)\sqrt{\frac{Q^2}{m^2}}\sqrt{\frac{M^2}{m^2}+\omega^2}\,\delta\left((1-\omega)(\alpha\omega+1)-\frac{m^2}{Q^2}\right),$$

(26e) $$\lim_{\text{Gen. Bj.}} \varrho_{TL^{(-)}} = 0\,.$$

From eqs. (25) and (26) we see that our original conclusions regarding scaling in the generalized Bjorken limits remain unchanged, and we conclude that for π^+ electroproduction computed in lowest-order perturbation theory the generalized Bjorken limit does not lead to scaling behavior.

4. – Discussion.

The loss of scaling invariance in the functions ϱ_3 and ϱ_4 (or ϱ_L and $\varrho_{TL^{(+)}}$) for the case of π^+ production can be traced directly to the term in the production amplitude corresponding to the second diagram of Fig. 2 *c*) in which the virtual photon interacts directly with the π^+. This diagram, which corresponds kinematically to replacing the u-channel proton poles of the σ and π^0 production amplitudes by a t-channel π^+ pole, cannot be argued away. Its presence is dictated by the general requirement of gauge invariance, which requires that the virtual photon interact with every charged particle which participates with the overall process. The origin of this difficulty does not actually stem from lowest-order perturbation theory; it is independent of perturbation theory. The models we considered are formally scale invariant if all masses are put to zero. Difficulties only arise if the theory is then singular, which is well known to be the case for diagrams with closed loops (8,9). However the present difficulty is due to another type of singularity, namely the $\varrho_{\mu\nu}$ has poles which touch the physical region if the masses are zero:

(27) $$\varrho_{\mu\nu} = \frac{a_{\mu\nu}}{s} + \frac{b_{\mu\nu}}{t} + \frac{c_{\mu\nu}}{u} + d_{\mu\nu}\,,$$

where (3)

$$s=(p+q)^2\,,\qquad t=(p'-q)^2\,,\qquad u=(p'-p)^2\,.$$

(8) A. I. VAINSHTEIN and B. L. IOFFE: *Zurn. Eksp. Teor. Fiz., Pisma Redakt.*, **6**, 917 (1967), English translation: *JETP Lett.*, **6**, 341 (1967); S. ADLER and W. K. TUNG: *Phys. Rev. Lett.*, **22**, 978 (1969); R. JACKIW and G. PREPARATA: *Phys. Rev. Lett.*, **22**, 975 (1969).

(9) SHAU-JIN CHANG and P. M. FISHBANE: *Phys. Rev. Lett.*, **24**, 847 (1970).

s, t and u are related to ν, ν_1 and $\varkappa$ in the following way:

$$(28)\qquad \begin{cases} s = M^2 - Q^2 + 2\nu\,, \\ t = m^2 - Q^2 - 2\nu_1\,, \\ u = m^2 + M^2 - 2\varkappa\,. \end{cases}$$

These singularities in $\varrho_{\mu\nu}$ forced us to retain mass terms which break scale invariances as $s \to s_{\min}$. (The problem in eqs. (19) and (26) are virtually the same; in (26) the singularity can be avoided by retaining mass terms, and (19) can be written in a scale-invariant but singular form.)

Trouble resulted from these singularities because we were taking the generalized Bjorken limit which is characterized by

$$\frac{2\nu}{Q^2} = -\frac{\varkappa}{\nu_1}\,,$$

which implies (in the limit) a factor $\delta(-st/Q^2 - p_\perp^2)$ and gives the problem when $p_\perp^2$ is finite. Thus this particular generalized Bjorken limit is not one in which scale invariance can sensib'y be expected. But the ordinary Bjorken limit may hold, as KÖGERLER and MURADYAN ([3]) have proposed.

The ordinary Bjorken limit is defined as

$$(29)\qquad \begin{cases} \nu,\; Q^2,\; -\nu_1,\; \varkappa \to \infty\,, \\ \dfrac{2\nu}{Q^2} = \text{fixed}\,, \qquad \dfrac{\nu_1}{\nu} = \text{fixed}\,, \qquad \dfrac{\varkappa}{\nu_1} = \text{fixed}\,, \end{cases}$$

and following ref. ([3]) we define besides the dimensionless variables ω and α (eqs. (16)) a third one

$$(30)\qquad \beta = 1 + \frac{2}{\omega}\frac{\varkappa}{\nu_1}\,.$$

Now eqs. (17) take the following form

$$(31a)\qquad \lim_{\text{Bj.}} \nu\varrho_1 = \overline{F}_1(\omega, \alpha, \beta)\,,$$

$$(31b)\qquad \lim_{\text{Bj.}} \nu^2\varrho_a = \overline{F}_a(\omega, \alpha, \beta)\,, \qquad a = 2, \ldots, 5\,,$$

with all the F_i dimensionless functions of ω, α and β. We now evaluate again the left-hand side of eqs. (31) using eqs. (8)-(10) and we indeed find that now all structure functions scale, in particular the structure functions $\nu^2\varrho_3$ and $\nu^2\varrho_4$

for the π^+ case, eq. (19):

$$\lim_{\mathrm{BJ}} \nu^2 \varrho_3 = \frac{4\pi g^2 \omega}{(1-\omega)(1+\alpha\omega)}\, \delta\left((\omega-1)(1+\alpha\omega) - \alpha\omega^2 \frac{\beta+1}{2}\right), \tag{32}$$

$$\lim_{\mathrm{BJ}} \nu^2 \varrho_4 = \frac{\pi g^2 \omega^2}{(\omega-1)(1+\alpha\omega)}\, \delta\left((\omega-1)(1+\alpha\omega) - \alpha\omega^2 \frac{\beta+1}{2}\right). \tag{33}$$

Furthermore we have also checked that the structure functions of KÖGERLER and MURADYAN ϱ_a, $a = T_1, T_2, L_1, TL^{\pm}$ assume their proposed scale-invariant form in the ordinary Bjorken limit. From eqs. (32) and (33) we find that only in the limit $\beta = -1$, which corresponds to the generalized Bjorken limit, do we again have trouble. This was to be expected.

Further, due to this pole at $\beta = -1$, we expect that the functions ϱ_3 and ϱ_4 dominate the other ones. However this has to be discussed in more detail and beyond lowest-order perturbation theory.

The most serious consequence of the breakdown of scaling in lowest-order perturbation theory is that it rules out the possibility of describing a semi-inclusive process by a simple parton model which will lead to scaling behavior in the *generalized Bjorken limit*. The parton model consists essentially of taking an incoherent sum of lowest-order perturbation theory amplitudes, and its ability to predict scaling in the ordinary Bjorken limit rests upon the scaling invariance of the lowest-order amplitudes. In the present context, a straightforward application of the parton model would produce an incoherent sum of nonscaling invariant terms, and it is exceedingly difficult to envision a mechanism which would cause this sum to exhibit a scaling behavior possessed by none of its terms.

We should also point out that the parton model faces a serious technical difficulty when applied to our processes. It is easy to see from eqs. (18), (19), (25) and (26) that in the generalized Bjorken limit and in the ordinary Bjorken limit some of the structure functions are more singular than a δ-function at the point $\omega = 1$, *i.e.* they have the behavior

$$\varrho(\omega) \sim \frac{\delta(\omega-1)}{\omega-1}. \tag{34}$$

In order to construct the parton model sum, the standard procedure is to define the total structure function to be

$$\left\{\begin{aligned} &\varrho^{T}(\omega) = \int_0^1 f(x)\, \varrho(x\omega)\, \mathrm{d}x, \\ &\int_0^1 f(x)\, \mathrm{d}x = 1, \end{aligned}\right. \tag{35}$$

and it is obvious from eq. (34) that the integral in eq. (35) does not exist. Hence, a more sophisticated procedure for performing the incoherent sum must be developed if a parton model of semi-inclusive processes is to be constructed.

Concerning the claims of ref. (4) we want to point out the following two differences in comparison with our approach:

Firstly, a spatial cut-off is introduced. (This assures that the singularities of ϱ_i stay away from the physical region.)

Secondly, an integration over the azimuthal angle is performed. Therefore their structure functions $\mathfrak{W}_i$ are not easily comparable with our ϱ_i because the integral is not at all trivial. (We have tried it but did not succeed.)

Therefore whether these two differences really assure that the generalized Bjorken limit exists, we do not know. But in view of our findings, namely that the generalized Bjorken limit breaks down due to singularities which move into the physical region, we remain doubtful concerning the results of ref. (4).

On the other hand we believe that the ordinary Bjorken limit may exist for semi-inclusive processes and may lead to scaling behavior in this case also (10).

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(10) N. F. BALI, L. S. BROWN, R. D. PECCEI and A. PIGNOTTI: *Phys. Rev. Lett.*, **25**, 557 (1970).

● RIASSUNTO (*)

Si esaminano le sezioni d'urto per l'elettroproduzione del mesone secondo due modelli di teoria di campo. Si trova che per l'interazione di mesoni scalari coi nucleoni le funzioni di struttura scalano nel limite di Bjorken generalizzato, ma che nel caso di un'interazione indipendente dalla carica mesone pseudoscalare-nucleone il fenomeno non si verifica sempre. Si cerca di spiegare l'origine di questa assenza e il suo significato per i modelli a partoni dello scattering altamente anelastico elettrone-adrone.

(*) *Traduzione a cura della Redazione.*

Нарушение инвариантности подобия в обобщенном пределе Бьёркена для теории возмущений.

Резюме (*). — Мы исследуем поперечные сечения электророждения мезонов в двух моделях теории поля. Мы находим, что для взаимодействия нейтральных скалярных мезонов с нуклонами структурные функции обнаруживают подобие в *обобщенном* пределе Бьёркена. Но подобие не всегда имеет место для случая зарядово-независимого взаимодействия псевдоскалярных мезонов с нуклонами. Мы объясняем происхождение этого нарушения и его значение для моделей глубоко неупругого электрон-адронного рассеяния.

(*) *Переведено редакцией.*