SIMPLE RELATION BETWEEN THE BARE AND PHYSICAL MASS OF FERMIONS ‡

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Results obtained in various field-theoretic models of electrodynamics and pseudoscalar meson-nucleon theory with massless pions strongly suggest that the bare mass M_0 of the fermion involved is simply related to its physical mass M by $M_0 = Z_2 M$. As a possible consequence of this relation it is suggested that the baryons as well as the electron and muon can be thought of as composite objects.

Motivated by the recent growing interest [1] in the application of combined scale and chiral invariance to strong interactions we were led to study the necessary conditions [2] for these symmetries in the framework of quantum field theory. On the basis of the Kamefuchi-Källen-Lehmann representation it was conjectured that these conditions - namely the vanishing of the bare masses - are related to the vanishing of the wave function renormalization constants.

In this letter we present the results of some explicit calculations which strongly suggest that in field theories with Yukawa coupling, in the limit of vanishing boson mass - and only if the limit exists - the following relation holds:

$$M_0 = Z_2 M$$

or (1)

$$\delta M \equiv M - M_0 = M(1 - Z_2),$$

where respectively M and M_0 are the physical and bare masses and Z_2 is the wave function renormalization constant of the fermion involved. For electrodynamics (QED) eq. (1) holds in the Fried-Yennie [3] (FY) gauge and in the radiation gauge in a specific frame, as it turns out, because these gauges are free of infrared divergences. On the other hand it is well known that δM^{QED} is gauge invariant. Therefore Z_2^{QED} has to assume a special gauge invariant value as discussed below.

To support our main result (eq. (1)) we have derived it exactly in lowest order and for the leading terms in *all* orders of perturbation theory for A) pseudoscalar meson nucleon field theory with massless neutral pions and B) FY-electrodynamics. We have also derived it in the Zachariasen-Thirring model [4]. The details of the calculation will be published elsewhere.

As is well-known, the bare mass of a fermion can be represented in the following way [5]:

$$M_0 = \int_{-\infty}^{+\infty} \mathrm{d}a\, a\, r(a)$$

or (2)

$$M_0 = M Z_2 + \int_M^{\infty} \mathrm{d}a\, a[s(a) - s(-a)],$$

where $r(a)$ and $s(a)$ are understood to be unrenormalized. The existence of the integral is assumed which amounts to introducing a cutoff if necessary. Eq. (1) can be fulfilled nontrivially if and only if

$$s(a) = s(-a). \qquad (3)$$

A) Pseudoscalar meson nucleon theory with vanishing meson mass.

(a) In lowest order one obtains

$$s^{(2)} = (g^2/32\pi^2)\frac{a^2 - M^2}{|a|a^2} > 0, \qquad (4)$$

from which eq. (1) follows.

(b) Given $s^{(2)}(a)$, the Lagrangian of the Zachariasen-Thirring [4] model can be easily written down

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$$\mathcal{L} = i\bar{\psi}(x)\gamma^4\partial_\mu\psi(x) - M_0\bar{\psi}(x)\psi(x) + \int_{-\infty}^{+\infty} da[i\bar{\psi}(a)\gamma^\mu\partial_\mu\psi(a) - a\bar{\psi}(a)\psi(a)]\,\theta(a^2-M^2) - ig\bar{\psi}(x)\int_{-\infty}^{+\infty} da\,\psi(a)f(a),$$

where

$$f^2(a) = g^{-2}(a-M)^2 s^{(2)}(a)\,\theta(a^2-M^2). \quad (5)$$

From eq. (5) and the spectralrepresentation of the inverse fermion propagator one finds

$$M_0 = M + g^2\int_{-\infty}^{+\infty} da f^2(a)/(a-M) = MZ_2,$$

where

$$Z_2 = 1 - g^2\int_{-\infty}^{+\infty} da\left\{\frac{f^2(a)}{(a-M)^2} + \frac{f^2(-a)}{(a+M)^2}\right\}. \quad (6)$$

(c) Now we turn to the calculation of δM and Z_2 in higher orders of perturbation theory. Eq. (1) was checked directly to order g^4 for the leading logarithmic term. After a long and tedious calculation we found

$$Z_2^{(4)} = -(g^4/2(32\pi^2)^2)\log^2(\Lambda^2/M^2),$$
$$\delta M^{(4)} = -Z_2^{(4)}M, \quad (7)$$

Λ being a relativistic cutoff. Independently we have obtained eq. (7) by using the powerful methods of the renormalization group, thereby heavily relying on the remarkable work of Astaud and Jouvet [6, 7]. In fact, using their results we are in the position to extend eq. (7) to all orders of perturbation theory. Namely, Jouvet and Astaud [6, 7] have shown that the leading terms of the vertex, the fermion self-energy and the vacuum polarization can be calculated to all orders, provided one knows the coefficients of the leading logarithmic terms of these functions in lowest order. In our case the coefficients are respectively given by $p_{11} = 1/16\pi^2$, $p_{21} = -1/32\pi^2$ and $p_{31} = -1/8\pi^2$. Since the coefficients of the leading term of $\delta M^{(2)}$ and $Z_2^{(2)}$ are equal, we obtain ($n = 1, 2 \ldots$):

$$Z_2^{(2n)} = -\delta M^{(2n)}/M = (g^{2n}/n!)\,p_{21}^n(-2+1)(-2.2+1)\ldots \times(-2(n-1)+1)\log^n(\Lambda^2/M^2). \quad (8)$$

B) FY-electrodynamics. In this case — as it turns out — one can literally take over eqs. (5) and (6) provided one makes the substitution $ig \to e$ and $1/32\pi^2 \to 3/16\pi^2$, thereby again establishing eq. (1). In a direct calculation we have obtained the following result for Z_2 to order e^4:

$$Z_2^{(4)} = (5e^4/2(16\pi^2)^2)\log^2(\Lambda^2/M^2) \quad (9)$$

which agrees with an independent calculation of $-(\delta M^{(4)}/M)$ [8]. Using the renormalization group one can again extend eq. (9) to all orders of perturbation theory. Now the coefficients p_{11}, p_{21}, p_{31} are given respectively by $-3/16\pi^2$, $-3/16\pi^2$ and $-1/12\pi^2$ and we finally obtain ($n = 1, 2 \ldots$)

$$Z_2^{(2n)} = -\delta M^{(2n)}/M = (e^{2n}/n!)\,p_{21}(-p_{31}+p_{21})(-2p_{31}+p_{21})\ldots \times(-(n-1)p_{31}+p_{21})\log^n(\Lambda^2/M^2). \quad (10)$$

Concerning radiation gauge electrodynamics several authors [9] pointed out that an infrared-divergence-free positive expression for the fermion propagator exists. Using their results, one easily establishes eq. (1) to lowest order. Higher order corrections are unknown but we conjecture that eq. (1) is true in radiation gauge, provided one chooses the right frame.

On the contrary, in a Yukawa theory with scalar coupling, eq. (1) is not likely to hold because of the well-known infrared divergences for vanishing boson mass. Indeed comparing the expression of Z_2, and δM in lowest order **

$$Z_{2\sigma}^{(2)} = 1 - (f^2/16\pi^2)\int_0^{\Lambda^2} dL \times\int_0^1 \frac{dz\,z^2[-M^2(1-z)^2 + 4M^2(1-z) + \mu_\sigma^2 z + Lz]}{[M^2(1-z)^2 + \mu_\sigma^2 z + Lz]^2};$$

$$\delta M_\sigma^{(2)} = -(f^2/16\pi^2)M\int_0^{\Lambda^2} dL \times \int_0^1 \frac{dz(1+z)}{M^2(1-z)^2 + \mu_\sigma^2 z + Lz}, \quad (11)$$

one readily sees that, while $Z_{2\sigma}^{(2)}$ is infrared divergent, $\delta M_\sigma^{(2)}$ is not. Again using the renormalization group, one can show that eq. (1) does not hold for the leading terms of $Z_{2\sigma}$ and δM_σ in all orders of perturbation theory.

** The subscript σ denotes the scalar meson contribution.

Let us now discuss the implication of our results:

1) Eqs. (6), (8) and (10) strongly suggest that eq. (1) is true in general, independent of any perturbation theory considerations.

2) In order to be true in general for QED eq. (1) can only hold in the form zero equals zero, i.e., $M_0^{QED} = Z_2^{QED} = 0$ (we exclude the trivial case $Z_2^{QED} = 1$). The value $Z_2^{QED} = 0$ is suggested by the fact that δM^{QED} is gauge invariant. Clearly in perturbation theory we have no convincing indication that Z_2^{QED} equals zero. Furthermore in gauges other than FY or radiation gauge eq. (1) does not even hold, basically due to infrared divergences. Therefore we expect $Z_2^{QED} = 0$ to be a property of an exact solution in the sense of providing an eigenvalue equation for the renormalized coupling constant [10]. (See point 5) below.)

3) Taking for granted that Z_2^{QED} equals zero we are naturally led to the composite particle condition [11] and this therefore suggests that not only strongly interacting particles but also leptons (with the possible exception of the neutrinos, because they have no direct electromagnetic interaction) can be thought of as composite objects and should show some structure at *very high* energies.

4) Due to the Ward identity the vertex renormalization constant Z_1^{QED} equals Z_2^{QED}. If Z_2^{QED} vanishes, Z_1^{QED} must also vanish. This conclusion agrees with that of other nonelectromagnetic models [12] of the vertex function renormalization constant. Furthermore, Källen's [13] result for the vertex function at large momentum transfer might after all be true, if $Z_1^{QED} = 0$, such that in this case there no longer arises any difficulty [14] due to gauge dependence of the result.

5) If $Z_2^{QED} = 0$, M_0^{QED} vanishes also [15]. In this case Astaud and Jouvet [6, 7, 16] have shown (without assuming $M = 0$) that Johnson's [17] result on the asymptotic behaviour of the vacuum polarization tensor, namely that it only diverges like a single logarithm, holds to all orders in perturbation theory. Also they showed that the unrenormalized change $e_0^2/4\pi \equiv \alpha_0$ has a fixed value α_{0f} in this case, determined * by the root of

$$H_0(M_0^2/k^2 = 0, \alpha_{0f}) \equiv \Psi_0(\alpha_{0f}) = 0. \qquad (12)$$

$H_0(\frac{1}{t}, \beta)$ — being identical to the famous Gell-Mann - Low function $\Psi(\beta)$ [18] — is the exact kernel of the integral equation for the photon propagator. There is then no implied contradiction [19] if we have α_0 fixed, since the renormalization charge α is fixed also: namely by $Z_2^{QED}(\alpha) = M_0^{QED}(\alpha) = 0$ [16].

6) Due to the equivalence theorem [20] we conjecture that eq. (1) also holds for the pseudovector coupling theory, especially because the limit of vanishing boson mass exists. Furthermore there is a deeper physical reason for the existence of this limit in pseudoscalar theories — in contrast to scalar theories as exhibited by eq. (11) — namely the conservation of chirality [21]. Although the simple pseudoscalar coupling theory has no chiral symmetry in the Lagrangian, it can acquire this symmetry if and only if $Z_2 = 0$ and $M \neq 0$ [2, 22]. Therefore also in γ_5 theories we come to the conclusion that $Z_2 = 0$, which is highly plausible in view of the success of the bootstrap hypothesis.

7) From the discussion above we learn that the bare mass of fermions (leptons and baryons) should actually vanish. This leads to the remarkable situation where the bare leptons and baryons cannot be distinguished † and to the possibility of an underlying fundamental matter field [24], which would represent these massless degenerate bare fermions.

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* Subscript zero denotes the cas $M_0^{QED} = 0$.

† See in this connection a similar observation by Nambu and Jona-Lasinio [23]

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