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Composite Particles in a Two-Dimensional Field Theory.

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In a two-dimensional model field theory ([1,2]) (one space-one time) we discuss the interaction of a vector field with zero bare mass (« electromagnetic » field) with a « fermion » field with bare mass m_0 in the limit $m_0 \to 0$. The case of $m_0 \neq 0$ has been recently studied by one of us ([3]).

It was shown some time ago ([4]) that if one calculates the boson self-mass ($\delta\mu^2 = \mu^2$) in second-order perturbation theory, the result one obtains in the limit $m_0 = 0$ is ambiguous. Specifically, one obtains

$$\mu^2 = \lim_{k^2 \to 0} \left(\frac{e^2}{\pi}\right) [1 - I(k^2, m_0^2)] \tag{1}$$

and two different results follow, depending on whether $k^2 \gtrless 4m_0^2$, namely,

$$k^2 < 4m_0^2\,, \qquad \mu^2 = 0 \tag{2}$$

$$k^2 > 4m_0^2\,, \qquad \mu^2 = \left(\frac{e^2}{\pi}\right) \qquad \text{for } m_0^2 = 0 \text{ exactly.} \tag{3}$$

It is this discontinuous behaviour exhibited by (2) and (3) we wish to consider in this note.

Let us recall first that eq. (3) is Schwinger's result, obtained in a nonperturbative calculation in a model in which m_0 is identically zero. As in our model m_0 is initially

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([1]) J. SCHWINGER: *Phys. Rev.*, **128**, 2425 (1962); *Theoretical Physics* (Vienna, 1963).

([2]) W. E. THIRRING and J. E. WESS: *Ann. of Phys.*, **27**, 331 (1964).

([3]) P. L. F. HABERLER: ICTP, Trieste, preprint IC/66/114.

([4]) E. MARX and I. SAAVEDRA: *Nucl. Phys.*, **60**, 337 (1964).

not zero, the second-order calculation need not be automatically correct (because there are two masses in the problem, e and m_0) and it has been pointed out ([5]) that therefore, higher-order terms might sum up to give the Schwinger result. In order to answer this question we quoted here the corresponding results for the fourth-order calculation. These are ([6])

$$k^2 < 4m_0^2\,, \qquad \varrho^{(4)}_{\mu\nu}(k)|_{k^2=0} = 0\,, \tag{4}$$

$$k^2 > 4m_0^2\,, \qquad \varrho^{(4)}_{\mu\nu}(k^2) = \frac{4m_0^2 e^4}{(2\pi)^4}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) F(k^2, m_0^2)\,, \tag{5}$$

where the function $F(k^2, m_0^2)$ diverges only logarithmically at the limit $m_0^2 \to 0$ and therefore $\varrho^{(4)}_{\mu\nu}$ vanishes in this limit.

Results (2) and (3) are therefore unaltered by this correction and this gives us some confidence to believe that the second-order results correspond to a genuine feature of the model considered and not to an accident due to the use of perturbation theory.

If we adopt this point of view, then we must try to understand this result, that is, we must try to see whether these two cases (vector bosons with masses zero and e^2/π, respectively) correspond to two objects with intrinsically different physical natures. In order to do this we recall that in field theory one can characterize a composite particle by the vanishing of the corresponding renormalization constant Z_3 ([7]). In our model, with $m_0 \neq 0$ one has, in second-order perturbation theory ([3]) ([*]),

$$Z_3 = \frac{1}{1 + e^2/6\pi m_0^2}\,. \tag{6}$$

Therefore the equation

$$Z_3(e^2, m_0^2) = 0 \tag{7}$$

has solutions $e^2 = \infty$ or $m_0^2 = 0$. Now, by definition e^2 is finite in this model; hence the only solution of eq. (7) is $m_0^2 = 0$ which corresponds exactly to the result (3). This particle we consider then as a composite particle. On the other hand, as in case (2) we cannot satisfy eq. (7), we consider the corresponding « photon » to be elementary. This is the distinction between the two cases.

Again it can be argued that our result holds only for second-order perturbation theory. However, as for dimensional reasons the higher corrections must be terms of the form $(e/m_0)^n$, it is really hard to see how this result could be altered by them. Moreover, as in electrodynamics Z_3 can in general be defined as the residue at the pole $k^2 = 0$ in the photon propagator ([8])

$$D_{\mu\nu}(k^2) \underset{k \to 0}{\simeq} -\frac{Z_3 g_{\mu\nu}}{k^2} + \text{gauge terms}\,, \tag{8}$$

([5]) S. DESER: *Math. Rev.*, **31**, 545 (1966).

([6]) P. L. F. HABERLER: Doctoral Dissertation, Vienna, 1966 (unpublished).

([7]) B. JOUVET: *Nuovo Cimento*, **5**, 1 (1957).

([*]) Z_3 was called B_4 in this reference.

([8]) See for instance, J. D. BJORKEN and S. D. DRELL: *Relativistic Quantum Fields* (New York, 1965), p. 304.

we can apply this definition to our model. For $m_0 = 0$ the exact « photon » propagator is given by [1,2]

$$D_{\mu\nu} = \frac{g_{\mu\nu}}{(e^2/\pi) - k^2} + \text{gauge terms}, \tag{9}$$

which does not exhibit a pole at $k^2 = 0$. Therefore we can argue that the renormalization constant has vanished.

Finally, let us remark that although in this particular model the « physical photon » $(\mu^2 = 0)$ cannot be considered as a composite particle (eq. (2)), if in eq. (3) we now let $e^2 \to 0$, then $\mu^2 \to 0$ and we have a composite vector boson with zero mass. This « photon » corresponds then to a bound state of uncharged massless fermions—a « neutrino » theory of light.

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