IL NUOVO CIMENTO VOL. XLVII A, N. 4 21 Febbraio 1967

Relation between Bare and Physical Mass of Vector Bosons in a Two-Dimensional Field Theory.

P. L. F. HABERLER (*)

International Atomic Energy Agency
International Centre for Theoretical Physics - Trieste

(ricevuto il 9 Gennaio 1967)

Several authors (1-4) have pointed out that Johnson's (5) proof concerning the relationship between the physical and bare mass of vector mesons might not be valid in general. The purpose of this note is to re-examine the problem in the framework of a two-dimensional model (6), which is an extension of the Schwinger-Thirring-Wess models (7); more specifically, the discussion will be carried out in a one space-one time dimensional theory of vector mesons φ_μ, with bare mass μ_0 in interaction with a fermion field, with bare mass m_0. The vector meson is coupled to a conserved gauge-invariant current. The limiting case as μ_0 tends to zero is compared with electrodynamics.

As is well known, starting from Dyson's equation one obtains (**) for the photon propagator $D_{\mu\nu}$ in the Landau gauge (3,8)

$$D_{\mu\nu}=\left(g_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)\frac{1}{-k^2-k^2\varrho(k^2)}\,. \tag{1}$$

(*) On leave of absence from Institute for Theoretical Physics, University of Vienna, Vienna.

(1) J. SCHWINGER: *Phys. Rev.*, **125**, 397 (1962).

(2) D. G. BOULWARE and W. GILBERT: *Phys. Rev.*, **126**, 1563 (1962).

(3) B. ZUMINO: *1962 Eastern Theoretical Physics Conference*, edited by M. E. ROSE (New York, 1963).

(4) B. ZUMINO: *Lectures on gauge invariance and mass of vector bosons*, Matscience report 32 (1965).

(5) K. JOHNSON: *Nucl. Phys.*, **25**, 435 (1961).

(6) P. L. F. HABERLER: to be published in *Acta Phys. Austriaca.*

(7) J. SCHWINGER: *Phys. Rev.*, **128**, 2425 (1962); W. THIRRING, J. WESS and F. SCHWABL: *A refined solution of two-dimensional model theories*, ICTP, Trieste, preprint IC/66/96; and numerous other authors. See, for example, A. S. WIGHTMAN: *Introduction to some aspects of the relativistic dynamics of quantized fields*, lectures given at the French Summer School of Theoretical Physics, Cargèse, Corsica, July 1964.

(**) We use relativistic units $\hbar=c=1$, $g_{00}=-g_{11}=1$; the coupling constant e has the dimension of a mass.

(8) B. ZUMINO: *Phys. Lett.*, **10**, 224 (1964).

We restrict our discussion to the case $\varrho(k^2)=\varrho^{(2)}(k^2)$, where $\varrho^{(2)}(k^2)$ is given by [6]

$$\varrho^{(2)}(k^2)=\int\limits_0^\infty \frac{ds^2\,\sqcap(s^2)}{s^2-k^2-i\varepsilon}\,,\qquad \sqcap(s^2)=\frac{2m_0^2e^2\,\theta(s^2-4m_0^2)}{\pi s^4\sqrt{1-4m_0^2/s^2}}\,. \tag{2}$$

A gauge-invariant result is obtained for the polarization tensor $\varrho_{\mu\nu}$ using the explicit dependence of the current on the external field [8].

Inspection of the poles of the function $D=(-k^2-k^2\varrho^{(2)}(k^2))^{-1}$ shows that *a)* $m_0<\sqrt{(1/2\pi)}\,e$, two vector particles exist, with physical masses $\mu_1=0$, stable, $\mu_2>2m_0$, unstable, and *b)* $m_0>\sqrt{(1/2\pi)}\,e$, a stable photon, $\mu=0$, exists. It follows that the spectral representation for $D_{\mu\nu}(k^2)$ reads [9]

$$\left\{\begin{aligned}&D_{\mu\nu}(k^2)=\left(g_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)\int\limits_0^\infty\frac{ds^2B(s^2)}{s^2-k^2-i\varepsilon}\,,\qquad B(s^2)=B_0\delta(s^2)+B_1(s^2)\,,\\&B_0=\frac{1}{1+e^2/6m_0^2\pi}\,,\qquad B_1(s^2)=\frac{1}{\pi}\,\mathrm{Im}\,D(k^2=s^2)\,.\end{aligned}\right. \tag{3}$$

We see that the presence of massive fermions modifies considerably the dynamical properties of the system. The physical photon has vanishing mass as required by the real physical world. At the limit $m_0\to 0$ ($B_0\to 0$) the Schwinger result [7,9] is obtained. Finally, in the strong-coupling limit a heavy, neutral, gauge-invariant vector boson appears: the existence of such a particle has been conjectured by LEITER [10].

The vector-meson propagator $\mathfrak{g}_{\mu\nu}(k^2)$ can be written in the following form:

$$\mathfrak{g}_{\mu\nu}(k^2)=\frac{g_{\mu\nu}-k_\mu k_\nu/k^2}{\mu_0^2-k^2-k^2\varrho(k^2)}+\frac{k_\mu k_\nu}{k^2\mu_0^2}\,, \tag{4}$$

where we assume $\varrho(k^2)=\varrho^{(2)}(k^2)$ and $\varrho^{(2)}(k^2)$ is given by (2). Inspection of the poles of $\mathfrak{g}=(\mu_0^2-k^2-k^2\varrho^{(2)}(k^2))^{-1}$ leads to the following results: 1) $\mu_0>2m_0$, $e/\sqrt{2\pi}\,m_0$ arbitrary, two particles appear, $\mu_1<2m_0$ stable, $\mu_2>2m_0$ unstable. 2) $0<\mu_0<2m_0$, *a)* $m_0<\sqrt{(1/2\pi)}\,e$, two particles, $\mu_1<\mu_0$ stable, $\mu_2>2m_0$ unstable, exist; *b)* $m_0>\sqrt{(1/2\pi)}\,e$, *α)* $\mu_0^2+2e^2/\pi<4m_0^2$, one stable particle, $\mu<\mu_0$; *β)* $4m_0^2-2e^2/\pi<\mu_0^2<4m_0^2$, two particles $\mu_1<\mu_0$ stable, $\mu_2>2m_0$ unstable, appear. The spectral representation, using the sum rule $\int\limits_0^\infty B(s^2)\,ds^2=1/\mu_0^2$ then reads

$$\mathfrak{g}_{\mu\nu}(k^2)=\int\limits_0^\infty\left(g_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)\frac{s^2H(s^2)}{s^2-k^2-i\varepsilon}+\frac{k_\mu k_\nu}{k^2\mu_0^2}\,, \tag{5}$$

---

(9) J. SCHWINGER: *Theoretical Physics* (Vienna, 1963), p. 85.
(10) D. LEITER: *Nuovo Cimento*, 30, 1245 (1963).

where

$$(6)\qquad \begin{cases} s^2H(s^2) = H_0\delta(s^2-\mu^2) + H_1(s^2)\,, & H_1(s^2) = \dfrac{1}{\pi}\operatorname{Im}\mathfrak{g}(k^2=s^2)\,, \\ \mu_0^2-\mu^2-\mu^2\varrho^{(2)}(\mu^2) = 0\,, & \mu^2 = \mu_0^2/(m_0^2)\,, \end{cases}$$

μ being the mass of a stable particle. The residue at the pole $k^2=\mu^2$ is given by $H_0 = 1/(1+\int\limits_0^\infty ds^2 s^2 \vdash\!\!\dashv(s^2)/(s^2-\mu^2)^2)$. The limiting case as μ_0 tends to zero can be performed in the usual way [2,4] by splitting φ_μ in a part tending to the electromagnetic field A_μ in the Coulomb gauge and another part tending to a free scalar field, if $\mu_0\to 0$. For the commutator of A_μ one finds

$$(7)\quad \langle 0|[A_\mu(x), A_\nu(x')]|0\rangle = i(g_\mu^\lambda+\partial_\mu a^\lambda)(g_{\nu\lambda}+\partial'_\nu a'_\lambda)\int\limits_0^\infty s^2H(s^2)\Delta(x-x', s)\,, \qquad a_\mu = \left(0, \frac{\partial_1}{\partial_1^2}\right).$$

It is readily established that μ vanishes as $\mu_0\to 0$, whence

$$(8)\qquad \delta(s^2-\mu^2)H_0 \xrightarrow[\mu_0\to 0]{} B_0\delta(s^2)\,, \qquad H_1(s^2)\xrightarrow[\mu_0\to 0]{} B_1(s^2)\,;$$

it may be noted that due to the particular behaviour of $\vdash\!\!\dashv(s^2)$, (2), the results of electrodynamics (3) have been smoothly obtained from (6).

In conclusion, it must be pointed out that, firstly, the problem of the relationship between bare and physical mass of vector mesons can only be answered by a more detailed investigation of the dynamics of the theory and this proved to be more involved, as one would expect; secondly, Johnson's relation seems to hold only for stable particles; thirdly, the physical situation depends in a nontrivial way on the magnitude of the coupling constant.

Although the above results were obtained in a two-dimensional theory utilizing lowest-order perturbation calculations, it is nevertheless gratifying that the theory exhibits some physically meaningful features which stimulate further investigation.

* * *

I am deeply indebted to Prof. C. Villi for numerous discussions and for critically reading the manuscript. The author would like to thank Profs. A. Salam and P. Budini, as well as the IAEA, for their kind hospitality at the International Centre for Theoretical Physics, Trieste, where this work was done. He also gratefully acknowledges a grant from UNESCO.